Candan *Advances in Difference Equations* 2013, **2013**:112
http://www.advancesindifferenceequations.com/content/2013/1/112
Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

Oscillation criteria for second-order nonlinear neutral dynamic equations with distributed deviating arguments on time scales

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Abstract

In this article, we establish some new oscillation criteria and give sufficient conditions to ensure that all solutions of nonlinear neutral dynamic equation of the form

$$\left(r(t)\left(\left(y(t)+p(t)y(\tau(t))\right)^{\Delta}\right)^{\gamma}\right)^{\Delta}+\int_a^b f\left(t,y\left(\delta(t,\xi)\right)\right)\Delta\xi=0$$

are oscillatory on a time scale $\mathbb{T}$, where $\gamma \geq 1$ is a quotient of odd positive integers.

Keywords: oscillation; dynamic equations; time scales; distributed deviating arguments

1 Introduction

The aim of this article is to develop some oscillation theorems for a second-order nonlinear neutral dynamic equation

$$\left(r(t)\left(\left(y(t)+p(t)y(\tau(t))\right)^{\Delta}\right)^{\gamma}\right)^{\Delta}+\int_a^b f\left(t,y\left(\delta(t,\xi)\right)\right)\Delta\xi=0 \tag{1}$$

on a time scale $\mathbb{T}$. Throughout this paper, it is assumed that $\gamma \geq 1$ is a quotient of odd positive integers, $0 < a < b$, $\tau(t): \mathbb{T} \to \mathbb{T}$, is rd-continuous function such that $\tau(t) \leq t$ and $\tau(t) \to \infty$ as $t \to \infty$, $\delta(t,\xi): \mathbb{T} \times [a,b] \to \mathbb{T}$ is rd-continuous function such that decreasing with respect to ξ, $\delta(t,\xi) \leq t$ for $\xi \in [a,b]$, $\delta(t,\xi) \to \infty$ as $t \to \infty$, $r(t) > 0$ and $0 \leq p(t) < 1$ are real valued rd-continuous functions defined on $\mathbb{T}$, $p(t)$ is increasing and

(H$_1$) $\int_{t_0}^{\infty} (\frac{1}{r(t)})^{\frac{1}{\gamma}} \Delta t = \infty$,

(H$_2$) $f: \mathbb{T} \times \mathbb{R} \to \mathbb{R}$ is a continuous function such that $uf(t,u) > 0$ for all $u \neq 0$ and there exists a positive function $q(t)$ defined on $\mathbb{T}$ such that $|f(t,u)| \geq q(t)|u^{\gamma}|$.

A nontrivial function $y(t)$ is said to be a solution of (1) if $y(t)+p(t)y(\tau(t)) \in C^1_{rd}[t_y,\infty]$ and $r(t)((y(t)+p(t)y(\tau(t)))^{\Delta})^{\gamma} \in C^1_{rd}[t_y,\infty]$ for $t_y \geq t_0$ and $y(t)$ satisfies equation (1) for $t_y \geq t_0$. A solution of (1), which is nontrivial for all large t, is called oscillatory if it has no last zero. Otherwise, a solution is called nonoscillatory.

We note that if $\mathbb{T} = \mathbb{R}$, we have $\sigma(t) = t$, $\mu(t) = 0$, $y^{\triangle}(t) = y'(t)$ and, therefore, (1) becomes a second-order neutral differential equation with distributed deviating arguments

$$\left(r(t)\left(\left(y(t) + p(t)y\left(\tau(t)\right)\right)'\right)^{\gamma}\right)' + \int_a^b f\left(t, y\left(\delta(t,\xi)\right)\right) d\xi = 0.$$

If $\mathbb{T} = \mathbb{N}$, we have $\sigma(t) = t + 1$, $\mu(t) = 1$, $y^{\triangle}(t) = \triangle y(t) = y(t+1) - y(t)$ and therefore (1) becomes a second-order neutral difference equation with distributed deviating arguments

$$\triangle\left(r(t)\left(\triangle\left(y(t) + p(t)y\left(\tau(t)\right)\right)\right)^{\gamma}\right) + \sum_{\xi=a}^{b-1} f\left(t, y\left(\delta(t,\xi)\right)\right) = 0$$

and if $\mathbb{T} = h\mathbb{N}$, $h > 0$, we have $\sigma(t) = t + h$, $\mu(t) = h$, $y^{\triangle}(t) = \triangle_h y(t) = \frac{y(t+h)-y(t)}{h}$ and, therefore, (1) becomes a second-order neutral difference equation with distributed deviating arguments

$$\triangle_h\left(r(t)\left(\triangle_h\left(y(t) + p(t)y\left(\tau(t)\right)\right)\right)^{\gamma}\right) + \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} f\left(t, y\left(\delta(t,kh)\right)\right)h = 0.$$

In recent years, there has been important research activity about the oscillatory behavior of second-order neutral differential, difference and dynamic equations. For example, Grace and Lalli [1] considered the following second-order neutral delay equation

$$\left(a(t)\left(x(t) + p(t)x(t-\tau)\right)'\right)' + q(t)f\left(x(t-\tau)\right) = 0, \quad t \geq t_0$$

and Graef *et al.* [2] considered the nonlinear second-order neutral delay equation

$$\left(y(t) + p(t)y\left(\tau(t)\right)\right)'' + q(t)f\left(y(t-\delta)\right) = 0, \quad t \geq t_0.$$

Recently, Agarwal *et al.* [3] considered second-order nonlinear neutral delay dynamic equation

$$\left(r(t)\left(\left(y(t) + p(t)y\left(\tau(t)\right)\right)^{\triangle}\right)^{\gamma}\right)^{\triangle} + f\left(t, y(t-\delta)\right) = 0. \tag{2}$$

Later, Saker [4] considered (2) but he used different technique to prove his results. In [5] and [6], the authors considered the second order neutral functional dynamic equation of the form

$$\left(r(t)\left(\left(y(t) + p(t)y\left(\tau(t)\right)\right)^{\triangle}\right)^{\gamma}\right)^{\triangle} + f\left(t, y\left(\delta(t)\right)\right) = 0,$$

which is more general than (2). For more papers related to oscillation of second-order nonlinear neutral delay dynamic equation on time scales, we refer the reader to [7–10]. For neutral equations with distributed deviating arguments, we refer the reader to the paper by Candan [11]. To the best of our knowledge, [12] is the only paper regarding to the distributed deviating arguments on time scales. The books [13, 14] gives time scale calculus and some applications.

2 Main results

Throughout the paper, we use the following notations for simplicity:

$$x(t) = y(t) + p(t)y\bigl(\tau(t)\bigr), \qquad x^{[1]} = r\bigl(x^{\triangle}\bigr)^{\gamma}, \qquad x^{[2]} = \bigl(x^{[1]}\bigr)^{\triangle} \tag{3}$$

and $\theta_1(t) = \delta(t,a)$ and $\theta_2(t) = \delta(t,b)$.

Theorem 2.1 *Assume that* (H_1) *and* (H_2) *hold. In addition, assume that* $r^{\triangle}(t) \geq 0$. *Then every solution of* (1) *oscillates if the inequality*

$$x^{[2]}(t) + A(t)x^{[1]}\bigl(\theta_1(t)\bigr) \leq 0, \tag{4}$$

where

$$A(t) = \frac{(b-a)q(t)(1-p(\theta_1(t)))^{\gamma}}{r(\theta_1(t))}\left(\frac{\theta_2(t)}{2}\right)^{\gamma}$$

has no eventually positive solution.

Proof Let $y(t)$ be a nonoscillatory solution of (1), without loss of generality, we assume that $y(t) > 0$ for $t \geq t_0$, then $y(\tau(t)) > 0$ and $y(\delta(t,\xi)) > 0$ for $t \geq t_1 > t_0$ and $b \geq \xi \geq a$. In the case when $y(t)$ is negative, the proof is similar. In view of (1), (H_2) and (3)

$$x^{[2]}(t) + \int_a^b q(t)y^{\gamma}\bigl(\delta(t,\xi)\bigr)\triangle\xi \leq 0 \tag{5}$$

for all $t \geq t_1$, and we see that $x^{[1]}(t)$ is an eventually decreasing function. We claim that $x^{[1]}(t) > 0$ eventually. Assume not then there exists a $t_2 \geq t_1$ such that $x^{[1]}(t_2) = c < 0$, then we have $x^{[1]}(t) \leq c$ for $t \geq t_2$ and it follows that

$$x^{\triangle}(t) \leq \left(\frac{c}{r(t)}\right)^{1/\gamma}. \tag{6}$$

Now integrating (6) from t_2 to t and using (H_1), we obtain

$$x(t) \leq x(t_2) + c^{1/\gamma}\int_{t_2}^{t}\left(\frac{1}{r(s)}\right)^{1/\gamma}\triangle s \to -\infty \quad \text{as } t \to \infty$$

which contradicts the fact that $x(t) > 0$ for all $t \geq t_0$. Hence, $x^{[1]}(t)$ is positive. Therefore, one sees that there is a $t_2 \geq t_1$ such that

$$x(t) > 0, \qquad x^{\triangle}(t) > 0, \qquad x^{[1]}(t) > 0, \qquad x^{[2]}(t) < 0, \quad t \geq t_2. \tag{7}$$

For $t \geq t_3 \geq t_2$, this implies that

$$y(t) \geq x(t) - p(t)x\bigl(\tau(t)\bigr) \geq \bigl(1 - p(t)\bigr)x(t)$$

then we conclude that

$$y^{\gamma}\big(\delta(t,\xi)\big) \geq \big(1-p\big(\delta(t,\xi)\big)\big)^{\gamma} x^{\gamma}\big(\delta(t,\xi)\big), \quad t \geq t_4 \geq t_3, \xi \in [a,b]. \tag{8}$$

Multiplying (8) by $q(t)$ and integrating both sides from a to b, we have

$$\int_a^b q(t) y^{\gamma}\big(\delta(t,\xi)\big)\triangle\xi \geq \int_a^b q(t)\big(1-p\big(\delta(t,\xi)\big)\big)^{\gamma} x^{\gamma}\big(\delta(t,\xi)\big)\triangle\xi. \tag{9}$$

Substituting (9) into (5), we obtain

$$x^{[2]}(t) + \int_a^b q(t)\big(1-p\big(\delta(t,\xi)\big)\big)^{\gamma} x^{\gamma}\big(\delta(t,\xi)\big)\triangle\xi \leq 0. \tag{10}$$

On the other hand, we can verify that $x^{\triangle\triangle}(t) \leq 0$ for $t \geq t_4$ and, therefore, we obtain

$$x(t) = x(t_4) + \int_{t_4}^t x^{\triangle}(s)\triangle s \geq (t-t_4)x^{\triangle}(t) \geq \frac{t}{2}x^{\triangle}(t), \quad t \geq t_5 \geq 2t_4.$$

From the last inequality, it can be easily seen that

$$x\big(\delta(t,\xi)\big) \geq \left(\frac{\delta(t,\xi)}{2}\right) x^{\triangle}\big(\delta(t,\xi)\big) \geq \left(\frac{\theta_2(t)}{2}\right) x^{\triangle}\big(\delta(t,\xi)\big), \quad t \geq t_6 \geq t_5, \xi \in [a,b].$$

Substituting the last inequality into (10), we have

$$x^{[2]}(t) + \int_a^b q(t)\big(1-p\big(\delta(t,\xi)\big)\big)^{\gamma} \left(\frac{\theta_2(t)}{2}\right)^{\gamma} \big(x^{\triangle}\big(\delta(t,\xi)\big)\big)^{\gamma}\triangle\xi \leq 0$$

and it can be found

$$x^{[2]}(t) + (b-a)q(t)\big(1-p\big(\theta_1(t)\big)\big)^{\gamma} \left(\frac{\theta_2(t)}{2}\right)^{\gamma} \big(x^{\triangle}\big(\theta_1(t)\big)\big)^{\gamma} \leq 0,$$

or

$$x^{[2]}(t) + \frac{(b-a)q(t)(1-p(\theta_1(t)))^{\gamma}}{r(\theta_1(t))} \left(\frac{\theta_2(t)}{2}\right)^{\gamma} x^{[1]}\big(\theta_1(t)\big) \leq 0,$$

which is the inequality (4). As a consequence of this, we have a contradiction and therefore every solution of (1) oscillates. □

Theorem 2.2 *Assume that* (H_1) *and* (H_2) *hold. In addition, assume that* $r^{\triangle}(t) \geq 0$, $\delta(t,\xi)$ *is increasing with respect to t and that the inequality*

$$\limsup_{t\to\infty} \int_{\theta_1(t)}^t A(s)\triangle s > 1 \tag{11}$$

holds. Then every solution of (1) *oscillates.*

Proof Let $y(t)$ be a nonoscillatory solution of (1). We can proceed as in the proof of Theorem 2.1 to get (4). Integrating (4) from $\theta_1(t)$ to t for sufficiently large t, we have

$$\begin{aligned}
0 &\geq \int_{\theta_1(t)}^{t} \bigl(x^{[2]}(s) + A(s)x^{[1]}\bigl(\theta_1(s)\bigr)\bigr)\triangle s \\
&= x^{[1]}(t) - x^{[1]}\bigl(\theta_1(t)\bigr) + \int_{\theta_1(t)}^{t} A(s)x^{[1]}\bigl(\theta_1(s)\bigr)\triangle s \\
&\geq x^{[1]}(t) - x^{[1]}\bigl(\theta_1(t)\bigr) + x^{[1]}\bigl(\theta_1(t)\bigr)\int_{\theta_1(t)}^{t} A(s)\triangle s \\
&= x^{[1]}(t) + x^{[1]}\bigl(\theta_1(t)\bigr)\left(\int_{\theta_1(t)}^{t} A(s)\triangle s - 1\right) > 0.
\end{aligned}$$

By making use of (11), we reach to a contradiction therefore the proof is complete. □

Theorem 2.3 *Assume that* (H_1) *and* (H_2) *hold. In addition, assume that* $r^{\triangle}(t) \geq 0$, $\delta(t,\xi)$ *is increasing with respect to t and there exists a positive rd-continuous* $\triangle$*-differentiable function* $\alpha(t)$ *such that*

$$\limsup_{t\to\infty} \int_{t_0}^{t} \left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{4\gamma(\frac{\theta_2(s)}{2})^{\gamma-1}\alpha(s)}\right)\triangle s = \infty, \tag{12}$$

where $(\alpha^{\triangle}(s))_+ = \max\{0, \alpha^{\triangle}(s)\}$ *and* $Q(s) = (b-a)q(s)(1-p(\theta_1(s)))^{\gamma}$. *Then every solution of* (1) *is oscillatory on* $[t_0,\infty)$.

Proof Suppose to the contrary that $y(t)$ is nonoscillatory solution of (1). We may assume without loss of generality that $y(t) > 0$ for $t \geq t_0$, then $y(\tau(t)) > 0$ and $y(\delta(t,\xi)) > 0$ for $t \geq t_1 > t_0$ and $b \geq \xi \geq a$. Proceeding as in the proof of Theorem 2.1, we obtain (7) and the inequality (10). Using (7) and Pötzsche's chain rule [15, Theorem 1], we obtain

$$\begin{aligned}
\bigl(x^{\gamma}(t)\bigr)^{\triangle} &= \gamma \int_0^1 \bigl[x(t) + h\mu(t)x^{\triangle}(t)\bigr]^{\gamma-1}\,dh x^{\triangle}(t) \\
&\geq \gamma \int_0^1 \bigl(x(t)\bigr)^{\gamma-1}\,dh x^{\triangle}(t) = \gamma\bigl(x(t)\bigr)^{\gamma-1}x^{\triangle}(t) > 0.
\end{aligned}\tag{13}$$

From (10) and (13), we obtain

$$x^{[2]}(t) \leq -(b-a)q(t)\bigl(1-p\bigl(\theta_1(t)\bigr)\bigr)^{\gamma}x^{\gamma}\bigl(\theta_2(t)\bigr) = -Q(t)x^{\gamma}\bigl(\theta_2(t)\bigr), \quad t \geq t_4. \tag{14}$$

Define the function

$$z(t) = \alpha(t)\frac{x^{[1]}(t)}{x^{\gamma}(\theta_2(t))}, \quad t \geq t_4. \tag{15}$$

It is obvious that $z(t) > 0$. Taking the derivative of $z(t)$, we see that

$$\begin{aligned}
z^{\triangle}(t) &= \bigl(x^{[1]}\bigr)^{\sigma}(t)\left(\frac{\alpha(t)}{x^{\gamma}(\theta_2(t))}\right)^{\triangle} + \frac{\alpha(t)}{x^{\gamma}(\theta_2(t))}x^{[2]}(t) \\
&= \frac{\alpha(t)x^{[2]}(t)}{x^{\gamma}(\theta_2(t))} + \bigl(x^{[1]}\bigr)^{\sigma}(t)\left(\frac{x^{\gamma}(\theta_2(t))\alpha^{\triangle}(t) - \alpha(t)(x^{\gamma}(\theta_2(t)))^{\triangle}}{x^{\gamma}(\theta_2(t))(x^{\sigma}(\theta_2(t)))^{\gamma}}\right).
\end{aligned}\tag{16}$$

Now using (14) in (16), we obtain

$$z^{\triangle}(t) \leq -\alpha(t)Q(t) + \frac{\alpha^{\triangle}(t)z^{\sigma}(t)}{\alpha^{\sigma}(t)} - \frac{\alpha(t)(x^{[1]})^{\sigma}(t)(x^{\gamma}(\theta_2(t)))^{\triangle}}{x^{\gamma}(\theta_2(t))(x^{\sigma}(\theta_2(t)))^{\gamma}}. \tag{17}$$

On the other hand, as in the proof of Theorem 2.1, it can be shown that for sufficiently large $t \geq t_5$

$$x(t) \geq \left(\frac{t}{2}\right)x^{\triangle}(t), \quad t \geq t_5 \geq 2t_4$$

and then

$$\gamma x^{\gamma-1}(t) \geq \gamma\left(\frac{t}{2}\right)^{\gamma-1}\left(x^{\triangle}(t)\right)^{\gamma-1}$$

or

$$\gamma x^{\gamma-1}\left(\theta_2(t)\right) \geq \gamma\left(\frac{\theta_2(t)}{2}\right)^{\gamma-1}\left(x^{\triangle}\left(\theta_2(t)\right)\right)^{\gamma-1}, \quad t \geq t_6 \geq t_5. \tag{18}$$

Since $x^{[2]}(t) < 0$, we have

$$x^{[1]}(t) > x^{[1]}\left(\sigma(t)\right). \tag{19}$$

Multiplying (18) by $x^{\triangle}(\theta_2(t))$ and using (19), it follows that

$$\begin{aligned} \gamma x^{\gamma-1}\left(\theta_2(t)\right)x^{\triangle}\left(\theta_2(t)\right) &\geq \gamma\left(\frac{\theta_2(t)}{2}\right)^{\gamma-1}\left(x^{\triangle}\left(\theta_2(t)\right)\right)^{\gamma} \\ &\geq \gamma\left(\frac{\theta_2(t)}{2}\right)^{\gamma-1}\frac{r(\theta_2(\sigma(t)))}{r(\theta_2(t))}\left(x^{\triangle}\left(\theta_2\left(\sigma(t)\right)\right)\right)^{\gamma} \\ &\geq \gamma\left(\frac{\theta_2(t)}{2}\right)^{\gamma-1}\frac{(x^{[1]})^{\sigma}(\theta_2(t))}{r(\theta_2(t))}. \end{aligned} \tag{20}$$

From (13), for sufficiently large $t \geq t_7 \geq t_6$, we have

$$\left(x^{\gamma}\left(\theta_2(t)\right)\right)^{\triangle} \geq \gamma x^{\gamma-1}\left(\theta_2(t)\right)x^{\triangle}\left(\theta_2(t)\right). \tag{21}$$

From (20) and (21), it follows that

$$\left(x^{\gamma}\left(\theta_2(t)\right)\right)^{\triangle} \geq \gamma\left(\frac{\theta_2(t)}{2}\right)^{\gamma-1}\frac{(x^{[1]})^{\sigma}(\theta_2(t))}{r(\theta_2(t))}. \tag{22}$$

Substituting (22) into (17), we obtain

$$z^{\triangle}(t) \leq -\alpha(t)Q(t) + \frac{\alpha^{\triangle}(t)z^{\sigma}(t)}{\alpha^{\sigma}(t)} - \frac{\gamma(\frac{\theta_2(t)}{2})^{\gamma-1}\alpha(t)}{(\alpha^{\sigma}(t))^2 r(\theta_2(t))}\left(z^{\sigma}(t)\right)^2.$$

Using the fact $u - mu^2 \leq \frac{1}{4m}$, $m > 0$, we have

$$\begin{aligned} z^{\triangle}(t) &\leq -\alpha(t)Q(t) + \frac{(\alpha^{\triangle}(t))_+}{\alpha^{\sigma}(t)}\left(z^{\sigma}(t) - \frac{\gamma(\frac{\theta_2(t)}{2})^{\gamma-1}\alpha(t)}{((\alpha^{\triangle}(t))_+)\alpha^{\sigma}(t)r(\theta_2(t))}\left(z^{\sigma}(t)\right)^2\right) \\ &\leq -\left(\alpha(t)Q(t) - \frac{((\alpha^{\triangle}(t))_+)^2 r(\theta_2(t))}{4\gamma(\frac{\theta_2(t)}{2})^{\gamma-1}\alpha(t)}\right). \end{aligned}$$

Integrating the last inequality from t_7 to t, we obtain

$$-z(t_7) < z(t) - z(t_7) \leq -\int_{t_7}^{t}\left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{4\gamma(\frac{\theta_2(s)}{2})^{\gamma-1}\alpha(s)}\right)\triangle s$$

or

$$z(t_7) > \int_{t_7}^{t}\left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{4\gamma(\frac{\theta_2(s)}{2})^{\gamma-1}\alpha(s)}\right)\triangle s$$

which contradicts (12). Therefore, the proof is complete. □

Theorem 2.4 *Assume that* (H_1) *and* (H_2) *hold and* $\sigma(t) \neq t$ *for each* $t \in \mathbb{T}$. *Let* $\alpha(t)$, $\delta(t,\xi)$, *and* $Q(s)$ *be as defined in Theorem* 2.3. *If*

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{2^{3-\gamma}(\mu(\theta_2(s)))^{\gamma-1}\alpha(s)}\right)\triangle s = \infty,$$

then every solution of (1) *is oscillatory on* $[t_0,\infty)$.

Proof Following the same lines as in the proof of Theorem 2.1, we get (7) and (10). Using the inequality,

$$x^{\gamma} - y^{\gamma} \geq 2^{1-\gamma}(x-y)^{\gamma}, \quad \gamma \geq 1,$$

we have

$$\begin{aligned} \left(x^{\gamma}(t)\right)^{\triangle} &= \frac{x^{\gamma}(\sigma(t)) - x^{\gamma}(t)}{\mu(t)} \geq 2^{1-\gamma}\frac{(x(\sigma(t)) - x(t))^{\gamma}}{\mu(t)} \\ &= 2^{1-\gamma}\left(\mu(t)\right)^{\gamma-1}\left(\frac{x(\sigma(t)) - x(t)}{\mu(t)}\right)^{\gamma} = 2^{1-\gamma}\left(\mu(t)\right)^{\gamma-1}\left(x^{\triangle}(t)\right)^{\gamma}. \end{aligned} \tag{23}$$

Now setting $z(t)$ by (15), using (17) and (23) we see that

$$z^{\triangle}(t) \leq -\alpha(t)Q(t) + \frac{(\alpha^{\triangle}(t))_+ z^{\sigma}(t)}{\alpha^{\sigma}(t)} - \frac{2^{1-\gamma}(\mu(\theta_2(t)))^{\gamma-1}\alpha(t)}{(\alpha^{\sigma}(t))^2 r(\theta_2(t))}\left(z^{\sigma}(t)\right)^2.$$

The remaining part of the proof is similar to that of Theorem 2.3, hence it is omitted. □

Example 2.5 Consider the following second-order neutral nonlinear dynamic equation

$$\left(\left(\left(y(t) + \left(\frac{t+a-1}{t+a}\right)y\left(\tau(t)\right)\right)^{\triangle}\right)^{5/3}\right)^{\triangle} + \int_{a}^{b} t^{-1/3}y(t-\xi)\triangle\xi = 0, \quad t \in \mathbb{T}$$

where $\gamma = \frac{5}{3}$, $r(t) = 1$, $p(t) = (\frac{t+a-1}{t+a})$, $q(t) = t^{-1/3}$. One can verify that the conditions of Theorem 2.3 are satisfied. Note that taking $\alpha(s) = s$, we see that

$$\limsup_{t\to\infty} \int_{t_0}^{t} \left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{4\gamma(\frac{\theta_2(s)}{2})^{\gamma-1}\alpha(s)} \right) \triangle s$$

$$= \limsup_{t\to\infty} \int_{t_0}^{t} \left((b-a)s^{-1} - \frac{1}{\frac{20}{3}(\frac{s-b}{2})^{2/3} s} \right) \triangle s = \infty.$$

Therefore, (1) is oscillatory.

Competing interests
The author declares that they have no competing interests.

Received: 26 April 2012 Accepted: 3 April 2013 Published: 18 April 2013

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doi:10.1186/1687-1847-2013-112
Cite this article as: Candan: **Oscillation criteria for second-order nonlinear neutral dynamic equations with distributed deviating arguments on time scales.** *Advances in Difference Equations* 2013 **2013**:112.